Nonprehensile Manipulation for Orienting Parts in the Plane *

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ABSTRACT

In [19] we presented a model of nonprehensile manipulation, using two one-degree-of-freedom palms. Under the assumptions of low friction and quasistatic motion, we developed a planning method for part reorientation with our model, starting from a known initial state. Our method finds feasible paths through the space of equivalent state configurations of the object in the palms, without requiring that the palms maintain stable support of the object over the entire path. We have shown that such a device can reliably orient parts in the plane. In this paper we extend our method to the case of reorienting a part to a desired goal from an unknown initial state. In addition to the all sliding contacts case which the model is based upon, we look at extensions to rolling contacts. We include the results of tests with example plans.

1 INTRODUCTION

In [19] we presented a model of nonprehensile manipulation, using two one-degree-of-freedom palms. Under the assumptions of low friction and quasistatic motion, we developed a planning method for part reorientation with our model. Our method finds feasible paths through the space of equivalent state configurations of the object in the palms, without requiring that the palms maintain stable support of the object over the entire path. We have shown that such a device can reliably orient parts in the plane. This device has demonstrated a number of points.

First, simple low degree of freedom devices can be used for the reliable manipulation of objects. In parts orienting scenarios, one would like to avoid complex mechanisms and sensors, which may break down or need careful recalibration. Sensory feedback is of course necessary in unknown or unstructured environments. However, for repetitive tasks in a structured environment, manipulators such as the one studied in this paper have a distinct advantage.

Second, because the devices are mechanically simple, the analysis of their mechanics is also relatively simple. To change the task from one object to another, or to change the goal state for the same object requires only a simple software modification. APOS trays or bowl feeders, which have the same strengths of reliable performance with low degrees of freedom, must be custom designed for each task, whereas devices such as this one can be used for a variety of tasks.

Third, by not relying on force closure grasps, we can exploit gravitational forces to guide the object into the correct state, without excessively precise control over the manipulator motions. Nor, as we will show in this paper, do we need extremely precise knowledge of frictional coefficients. Rough estimates are sufficient. The primary mechanical analysis used by the planner is frictionless and quasistatic. Knowledge of frictional and dynamic forces is only approximate, yet the resulting plans are robust to initial conditions, friction and to small errors in the calibration of the manipulator.

In this paper we extend the method presented in [19] to the case of reorienting a part to a desired goal from an unknown initial state. In addition to the all sliding contacts case which the model is based upon, we also look at extensions to rolling contacts. We illustrate the results with tests of example plans.

2 RELATED WORK

Systems which perform the type of tasks we will focus on include bowl feeders and Automatic Parts Orienting Systems ([3], [9]), where a large number of parts in arbitrary orientations are singulated and oriented by their interactions with (in the case of bowl feeders) fences and other obstacles, or (in the case of APOS systems) by an induced vibration and interaction with pallets of special shapes. Many of the techniques in the literature ([10], [12], [17], [2]) use the mechanics of pushing to design parts orienters or parts filters with a sequence of fences, similar in function to the vi-

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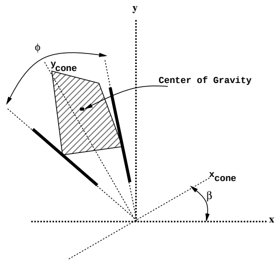


Figure 1: Cone frames

bratory bowl feeders and other orienting systems described by Boothroyd, *et.al.* [3]. Peshkin and others ([12], [17]), use fixed fences which interact with a part on a conveyer belt. Like bowl feeders and the APOS decive, this device is designed for a specific reorientation task, and is not reprogrammable. Mani and Wilson [10] use fixed fences and a programmable moving table. Akella, *et.al* [2] describe a device which uses a one degree of freedom arm, which interacts with a part on the conveyer belt. These last two devices are reprogrammable, and can in principle be used or reused for a variety of reorientation tasks.

There has also been much work on manipulation in the face of uncertainty. Brost [4] finds sets of actions which reliably orient a part in the presence of uncertainty in the part's location. Goldberg, and later Rao and Goldberg ([8], [13]) found algorithms for determining sequences of squeezes of a parallel jaw gripper which will reliably orient (up to symmetry) frictionless polygonal or algebraic planar parts from an arbitrary and unknown initial orientation, without sensors. Mason and Erdmann [7] use gravity to propel parts onto a flat surface, or into a corner formed by two perpendicular flat surfaces, in such a way that the resulting contact forces reliably orient a part.

The work described in [19] and in the present paper follows a similar method as that of Trinkle, Ram, Farahat and Stiller ([16], [14]). Their analysis and planning use the idea of *contact formations* originally presented by Desai, and incorporated into a planner for dextrous manipulation by Trinkle and Hunter [15]. Abell and Erdmann [1] use a similar method to study hand-offs of stably supported objects between sets of frictionless point fingers.

3 Frictionless Manipulation: Known Initial State

We briefly review the results of [19]. In that paper, we derived a method of planning reorientations of planar polygonal parts from known initial states to a desired goal, under the following assumptions. First, force balance is achieved by the palms stably supporting the object against gravity. No other external forces are considered, hence complete force/form closure is not necessary. Second, the motions of the manipulator are slow compared to gravity, so that the kinetic energy imparted to the object by the motion of the palm is dominated by the object's potential energy. Third, the contacts between the object and the palms are very low friction (the contacts are all sliding), so that we may approximate the system with a frictionless analysis. Fourth, we model the palm as a cone formed by two palms connected at a central hinge. This cone is formed by the intersection of the lines along which the palms lie. If we set a frame in the cone such that the y axis is the bisector of the cone, then all possible motions of the cone can be described by the variation of two parameters: the angle ϕ of the cone opening, and the orientation of the cone frame in the world, β.

In this paper, we will derive a method of planning reorientations from an unknown initial state, using the model given above. We will then show how to relax the all sliding contacts assumption to handle rolling contacts.

If we look, for the moment, at the object only after it has made contact with the cone, then the cone/object configuration can be characterized as a point in the space (θ, ϕ, β), where θ is the orientation of the object in the world frame. We call all stable resting configurations that correspond to a particular side of the object in edge contact with a particular palm *equivalent configurations*. A region of equivalent configurations, or an equivalence region, is planar and simply connected. It may be thought of as the bottom of an potential energy well in which the object state is trapped. In the terminology of [14], an equivalence region corresponds to a region in configuration space for which a completely (first order almost everywhere) stable path exists between any two points in the region.

For the frictionless, low kinetic energy case, almost every point on the configuration space constraint surface, except for those corresponding to unstable equilibrium orientations, is attracted to a unique stable resting configuration. Once the object is in a stable configuration it can be brought by a stable path to any other point in the same equivalence region. In particular, it is possible to bring the object to a state on the boundary of the equivalence region. If we then move the object state outside the equivalence region, the object will be attracted to another stable resting configuration, corresponding to a different equivalence region. This transition is reliable, even though the manipulator does not maintain stable support of the object during the entire transition, as long as the object's kinetic energy is low compared to its depth in the potential energy well.

In order to determine which orientations of a particular part can be brought to which other orientations, one must first determine all the equivalence regions (two for every flat face of the convex hull of the object) and their boundaries. Then, divide each boundary into segments, according to which new equivalence region the object will fall into

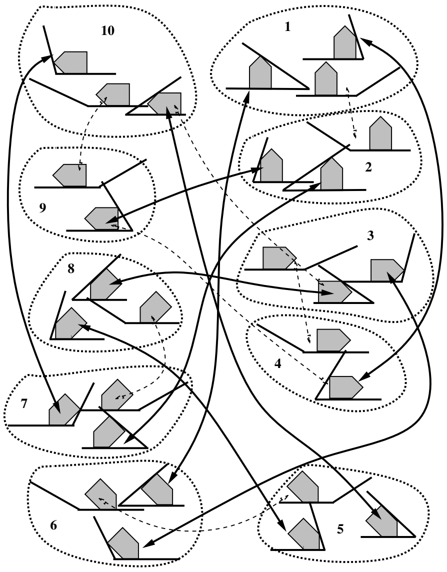


Figure 2: Transition graph for example object. Transition motions are made by pure tilts in the appropriate direction.

from that boundary segment. From these boundary segments, construct the graph $\mathcal{G}$ whose nodes are the equivalence regions, with arcs denoting which equivalence regions transit into another. Each arc is labeled with the appropriate set of cone configurations, and the direction in cone configuration space in which the cone must be moved. Figure 2 shows $\mathcal{G}$ for our example object.

The planning problem has now been segmented into two parts. Given the initial and desired final configurations of the system, the high level problem is how to get from the initial to the final equivalence region. We determine these paths by breadth-first search through $\mathcal{G}$. Once it has been established that a high level path exists, the lower level trajectory planning problem for each equivalence region (node) is to determine the trajectory which the cone must follow to reorient the part. To determine trajectories through equivalent regions, we can take advantage of the fact that equivalence regions are piecewise straight-line connected, as is shown in [18].

4 Planning from an Unknown Initial State

In many applications, such as parts feeding, the initial state of the object may not be known. In this section we will focus on the problem of determining a palm trajectory which will always bring the part to a single final state.

4.1 Frictionless case

Under the assumptions about the system used in the previous section, the transition from initial equivalence region to final equivalence region is unique for a given cone opening ϕ. Suppose we have a polygon with N statically stable edges. Then there are $2N$ equivalence regions, and we can enumerate the 2^{2N} elements of the *power set* of the equivalence regions: that is, the set of all possible combinations of the equivalence regions. For instance, if we have a set of equivalence regions $\{$A, B, C, D$\}$, then the power set of this set of equivalence regions would be $\{$ $\{$A$\}$, $\{$B$\}$, $\{$C$\}$, $\{$D$\}$, $\{$A, B$\}$, $\{$A, C$\}$, $\{$A, D$\}$, ... $\{$A, B, C, D$\}\}$. Given some set of palm motions, we can build a larger transition graph, $\mathcal{GG}$, where each node is an element of the power set, and each arc corresponds to a palm motion. Each arc then connects a set of initial states to its corresponding set of final states. For instance, suppose for a given palm motion, if the object was initially in state A, the motion will transfer the system state to C. If the object started in state B, the same palm motion will transfer the system to state D. Then the graph $\mathcal{GG}$ would include an arc between the nodes corresponding to the sets $\{$A$\}$ and $\{$C$\}$, an arc between the nodes corresponding to the sets $\{$B$\}$ and $\{$D$\}$, and an arc between the nodes corresponding to the sets $\{$A, B$\}$ and $\{$C, D$\}$. All these arcs would represent the same palm motion.

If we wish to find a manipulator trajectory which always terminates with the polygon in a particular final state, we can do a breadth-first backchaining search from our desired final state, hoping to find a path backwards to the set of all possible initial states. We will call such a sequence of arcs a *homing sequence*.

In practice, since an arc will generally correspond to bringing an object from resting on one palm to resting on the other palm, we do not have to consider all 2^{2N} elements of the power set. We will generally have to consider the set of all initial states, all combinations of equivalence regions corresponding to resting on the left palm, and all combinations of equivalence regions corresponding to resting on the right palm, for a total of $2^{N+1} - 1$ sets of combinations of states.

A planner to find homing sequences using the frictionless quasistatic assumption was written in C on a Decstation 5000/20. Figure 3 shows a homing sequence found for our example object. A set of cone openings, $\{\phi_i\}$, was selected, and for each ϕ_i, two motions were considered. One type of motion considered was to start with the left palm horizontal and then and tilt both palms clockwise, keeping their relative angle fixed, until the right palm is horizontal. The other type of motion was to start with the right palm horizontal, and tilt clockwise until the left palm is horizontal. It is possible, as shown in [18], to select the set $\{\phi_i\}$ such that its corresponding set of motions, as described above, satisfies the following condition. For any state to which the part can be homed using only pure tilt motions, a corresponding homing sequence can be constructed from this set of motions. Hence, the planner is complete: if a homing sequence exists, it will be found.

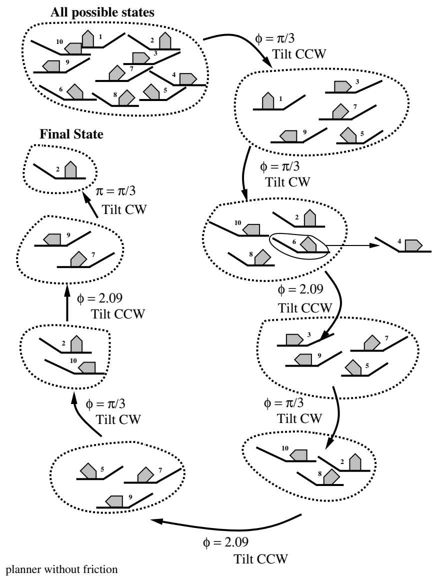


Figure 3: Example homing sequence. The circled configuration in the third stage would often collapse to another state, shown, which was not predicted by the planner.

Figure 4: Plastic cone manipulator used to test plans

In Figure 3, each arc is labeled with the ϕ used, and the direction of the tilt. Generating the graph $\mathcal{GG}$ generally took on the order of two or three minutes. Once the graph was generated, reorientation plans could be found (or determined not to exist) in about 10 seconds. The plans were tested on a plastic cone manipulator (Figure 4), mounted on a tilted air table to reduce support friction The above plan was one of the sequences found for the example object. However, when the plan was tried on the airtable system, it failed regularly. The problem was the frictional instability of the circled state in the third stage of the plan shown in Figure 3. The object would often roll off this edge, into another state not anticipated by the planner.

4.2 Friction

If one of the edge contacts rolls, rather than slides, a move of the palms which should be entirely stable, according to the frictionless analysis, could in fact cause the object to transit to another state. As has been shown, this was one of the reasons that the plans generated by the algorithm in the previous section would fail in practice.

We can apply techniques for planar frictional quasistatic analysis, based on those presented in [5] and [11], to the problem at hand.

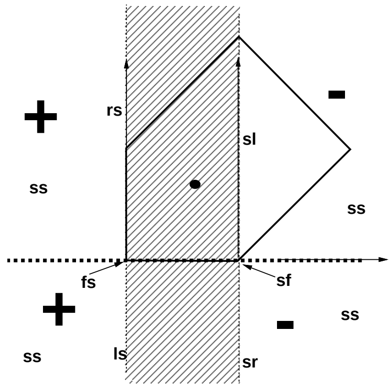


Figure 5: Regions of feasible centers of rotation, and their associated rotation senses and contact modes. The shaded region is a region of infeasibility. Each contact mode string gives the contact mode for the left and right contact points, respectively. “f” is a fixed or rolling contact, “l” means the contact point slides left, “r” means it slides right, and “s” means the contact breaks.

The contact forces on a body can be adequately represented, under the rigid body assumption, by only considering point contact forces; for edge-edge contact one need only consider the endpoints of the contact. Let $\mathbf{x}$ represent the position of the object in configuration space, F_A be the net applied force and torque on the object and let $\mathcal{C}_F$ be the cone of frictional contact forces exerted on the body. In the problems we will be solving, we will generally know $\mathcal{C}_F$ and F_A and will be hypothesizing object acceleration directions.

We can use the kinematic analysis due originally to Reuleaux (*The Kinematics of Machinery*, 1876) which is adapted to planar robotics problems and described in [5]. The method of Reuleaux represents a rigid planar motion by its center of rotation; it is easy to reverse the process and derive the configuration space motion vector from a center of rotation (or from a line of rotation centers). While in general, the center of rotation is not the same as the center of acceleration, under the quasistatic assumption, the instantaneous velocity of the part is low enough that the motion of the object will be dominated by the applied forces, and the two centers will coincide.

Using the technique described in [5], and also used in [11], we can find all kinematically feasible centers of rotation for the object motion (Figure 5). Briefly, at every contact point, if the contact normal points into the object, only counterclockwise centers of rotation are feasible to the left of the contact normal. Only clockwise centers of rotation are feasible to the right of the normal. Along the line through which the contact normal passes, either direction of rotation is feasible. By looking at every contact point, and eliminating regions where the contact points demand conflicting rotation senses, we can determine the regions of feasible rotation centers. We can also determine the contact mode of each contact point for a given center of rotation, and hence determine the components of $\mathcal{C}_F$. See [5] or [11] for more details.

Given a center and direction of rotation, one can easily determine the resulting motion of an object of interest in the plane. In the following, we will assume that the center of gravity (CG) of the object is the origin of our reference frame, and the radius of gyration is the unit of length.

Under the quasistatic assumption, the instantaneous acceleration can be adequately represented by

$$\begin{aligned} \mathbf{a} &= -\alpha \times \mathbf{r} \\ \begin{bmatrix} a_x \\ a_y \\ 0 \end{bmatrix} &= -\begin{bmatrix} 0 \\ 0 \\ \alpha \end{bmatrix} \times \begin{bmatrix} r_x \\ r_y \\ 0 \end{bmatrix} \end{aligned}$$

where $\mathbf{a}$ is the acceleration of the center of gravity due to some rigid motion of the plane, $\mathbf{r}$ is the center of rotation of that rigid motion with respect to the object's center of gravity, and α is the rotation of the object. $\times$ denotes the cross product. This gives the motion of the object's CG in the plane, and hence the first two components of the configuration space acceleration vector. The third component would of course be α.

Hence, given a center of rotation, $\mathbf{r}$, and a direction of rotation, $\operatorname{sgn} \alpha$, the direction of the resulting configuration space acceleration is given by

$$m\ddot{\mathbf{x}} \propto \begin{bmatrix} r_y \\ -r_x \\ 1 \end{bmatrix} \operatorname{sgn} \alpha.$$

The problem of finding feasible motions of the object subject to frictional contact can now be stated:

Determine whether $m\ddot{\mathbf{x}}$ is contained in $\mathcal{C}_F \oplus F_A$,

where $m\ddot{\mathbf{x}}$ can be derived from a postulated center of rotation, and $\oplus$ denotes the operation of cone (nonnegative vector) combination. Problems of this form can be easily solved using, for example, linear programming.

4.3 When does it slide and when does it roll?

Referring to Figure 6, we wish to determine the behavior of the object on the palm under the influence of gravity. For what orientations of the palm will the object remain stationary? For orientations of the palm steeper than that, will the object slide down the palm, or roll about a vertex?

Suppose we are tilting the palm clockwise. Then the critical contact is the right vertex of the resting edge. Let h be the height of the center of gravity, and w_r be the tangential distance from the center of gravity to the right vertex. Consider the case shown in Figure 6, where the friction cone at the right contact does not contain the center of gravity. It can be shown, using the technique described in the previous section, that if the magnitude of the palm's tilt angle is less than $\arctan \mu$, the object will stick, as expected. For steeper tilt angles, the object will slide without rolling. Informally, the object will not rotate because the contact force at the right vertex always points to the right of the center of gravity, exerting a counterclockwise moment about the center of gravity, which will be balanced by the moment due to the contact force at the left vertex.

Now consider the case shown in Figure 7, where the right friction cone contains the center of gravity. In this case, it can be shown that if the palm's clockwise orientation β is less than $\arctan \frac{w_r}{h}$, the object will stick, and for steeper tilt angles, the object will roll clockwise about the right vertex. This is because when $\beta < \arctan \frac{w_r}{h}$, the gravity vector

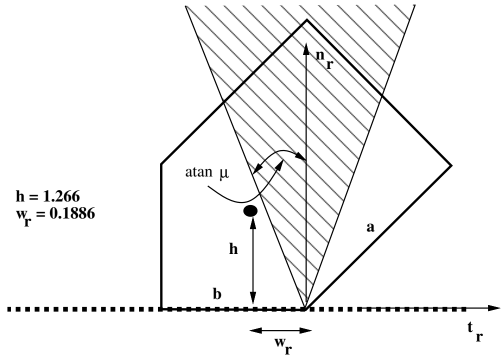


Figure 6: CG is outside of friction cone ($\mu < 0.154$). Edge b will either stick or slide without rolling for clockwise rotations of the palm.

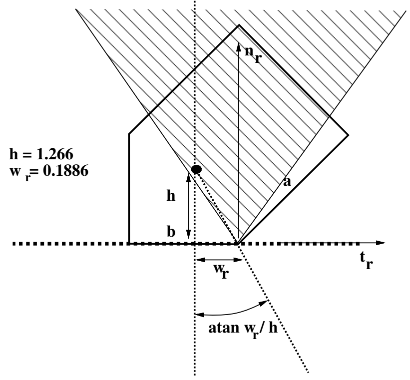


Figure 7: Friction cone contains CG ($\mu > 0.154$). Edge b will either remain stationary or rotate about its right vertex for clockwise rotations of the palm.

points to the left of the right vertex, and so the compensating contact force at the right vertex will point to the right of the center of gravity. This contact force will exert a counterclockwise moment about the center of gravity, which will be balanced by the moment due to the contact force at the left vertex. Hence the object will not rotate. Because the applied forces at both contacts will be inside the friction cone, the object will stick, as well. For steeper tilt angles, gravity will point to the right of the right vertex, and the compensating force will point to the left of the center of gravity. This will exert a clockwise moment about the center of gravity, which cannot be balanced by the moment due to the contact force at the left vertex. Hence, the object will rotate clockwise about the right vertex.

The above can be summarized as follows:

Consider an object resting on stable edge contact with a single palm. Let $\mu_{crit} = \frac{w_r}{h}$, and $\alpha_{crit} = \arctan \mu_{crit}$. When the palm is tilted clockwise, to an angle of magnitude β, one of the following cases occurs. If $\mu < \mu_{crit}$, the object will stick if $\beta < \arctan \mu$. For steeper tilt angles, the object will slide without rotating. If $\mu > \mu_{crit}$, the object will stick if $\beta < \alpha_{crit}$, For steeper tilt angles, the object will rotate clockwise about the right vertex. Similar results hold for the left vertex when the palm is tilted counterclockwise.

Using this result, we now have a stability criterion for a particular equivalence class with respect to the contact friction of our system. If the equivalence class corresponds to resting on the left palm, we consider μ_{crit} for the right vertex of the resting edge. If the object is resting on the right palm, we consider μ_{crit} for the left vertex of the resting edge. In either case, if μ_{crit} is less than the system contact friction, the resting edge will have a tendency to roll as the palm tilts out of the horizontal, and the equivalence class may be considered to be frictionally unstable.

Frictional instabilities can easily be incorporated into the power set approach, with the additional assumption that the object will roll to the next stable edge and stay there, rather than tumble further beyond to the next stable edge. By knowing which states are frictionally unstable, we can identify the additional states which may result from a given palm motion. For example, in the configuration shown in Figure 7, $h = 1.226$ and $w_r = 0.1886$, hence $\mu_{crit} = 0.154$. If the system coefficient of friction $\mu_0 > \mu_{crit}$, then we know that when the polygon is resting on the left palm on edge b it will have a tendency to roll over the right vertex, onto edge a. Note that this example is the symmetric counterpart to the problem case in Figure 3.

While generating the transition graph for the power set of the object states, we take both $(a$, left) and $(b$, left) as possible end states for motions which the frictionless analysis says should result in a transition to state $(b$, left). This covers both the possibilities that the object will either slide or

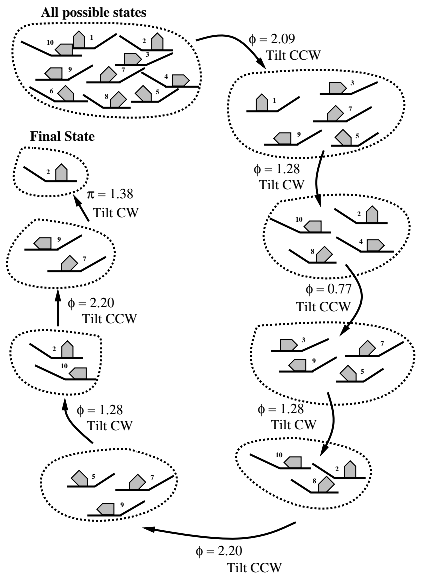


Figure 8: Example homing sequence

roll. We can then search this graph for a homing sequence. Because we incorporate the possibility of both sliding and rolling contacts, the transition graph and any resulting homing sequences will be valid for any coefficient of friction from 0 to μ_0. For frictional coefficients higher than μ_0, the graph may no longer be valid, since edges which were assumed to always slide may become frictionally unstable.

The planner was extended to take frictional instability under consideration. Figure 8 shows a homing sequence found for our example object, using the set of cone openings shown to build the arcs. The coefficient of friction, μ_0, was taken to be about 0.2. The sequence was run with the object started in all ten of the possible starting conditions, and was successfully brought to the goal state from each initial state. Then the object was dropped into the palms, into an arbitrary initial state, and the sequence was executed. Out of 15 such trials, the object failed to reach the desired goal state once.

The planner attempted to find homing sequences to home the example object to all possible final states. However, it could only find sequences for four of the possible ten final states, even though only two of the edges of the object had a frictional instability for a coefficient of friction of 0.2. Each of these sequences was tested, as above. The results are summarized in Table 1.

Although orientation plans for the other six resting states of the object were not found, those which were found were quite repeatable, as shown in Table 1. In principle, orientation plans for more resting states can probably by found by setting a tighter bound on the friction parameter used by the system (specifying an exact coefficient of friction, or a minimum as well as a maximum bound). A tighter

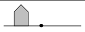
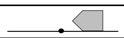
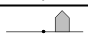
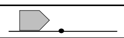
Goal State	(# successes)/(# trials)
	14/15
	12/15
	14/15
	15/15

Table 1: Trials of homing sequences

bound would eliminate some of the uncertainty in the outcome of certain motions in our model. However, assuming an exact value for the coefficient of friction is a nontrivial assumption, which we believe will reduce the robustness of the resulting plans. For example, in the two palm manipulation system described in [6], a significant cause of plan failure was the slipping of a contact which was expected to roll, either because the coefficient of friction was not as the planner expected, or because of inaccuracies of the manipulators when executing a specified motion. This resulted in a general brittleness of the reorientation plans. Such brittleness can of course be offset by the use of sensors to detect slipping, which brings up a host of other complexities which are happily avoided by the system presented in the present paper.

This version of the planner can also be used to find frictionally reliable paths from known initial states as well as from unknown ones. Instead of backchaining all the way back to the set of all states, the backchaining terminates upon reaching a node which contains the desired initial state.

5 Conclusion

In [19] we presented a preliminary analysis of nonprehensile manipulation by two low friction palms, and developed a planning method for part reorientation with our model. In this paper, we have developed a method for finding manipulator trajectories which will reliably orient planar polygonal parts to a desired goal from an unknown initial state. We have also extended the model to account for rolling frictional contact. We have demonstrated a simple low degree of freedom device that can reliably be used for object manipulation in a structured task environment. This device can be easily reprogrammed to perform a variety of reorientations on different objects. The reorientation plans are robust to initial conditions, friction within predetermined tolerances, and to imprecise calibration of the manipulator. We believe that the continued development of devices such as the one presented in this paper is necessary to meet the demands of modern industrial automation.

6 Acknowledgements

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