Nonprehensile Two Palm Manipulation with Non-Equilibrium Transitions between Stable States *

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Abstract

Manipulation without prehension is a natural way of handling objects for both humans and machines. Nonprehensile operations are appropriate when complete constraint over the object to be manipulated is either undesirable or impractical, but some control over the object is desired over its entire trajectory, in order to bring the object reliably to a desired final state. Research to date has explored only a small portion of this class. We are interested in controlling the shape of the constraint surfaces so that constraint and external forces naturally attract the system to the desired state, even if the object momentarily loses stability during the motion. We present a preliminary analysis of the nonprehensile orientation of planar objects by two low friction palms joined at a central hinge. These palms support an object in a gravitational field, without grasping or gripping. We determine connected regions of stable states of the object, and give a method of planning part orientation based on a graph search over these regions, allowing non-equilibrium transitions between them. We conclude with the the results of simulations and tests of an example plan.

1 Introduction

Prehension may be defined as "The act of taking hold, seizing or grasping, as with the hand" (Webster's 3rd International Dictionary). *Nonprehensile manipulation*, then, can be defined as the manipulation of objects without grasping them. There is a large class of operations during which complete constraint over the object to be manipulated is either undesirable or impractical, but some control over the object is desired over its entire trajectory, in order to bring the object reliably to a desired final state. One example is moving an object which is too large to be grasped, but which must be pushed or rolled into position. Repeated manipulation of many identical objects, as in parts orienting, is another. In this case, the precise trajectory of each individual part isn't precisely known or controlled. It is only desired that the parts all end up in the identical goal state. We claim that nonprehensile manipulation is appropriate for exactly these type of tasks.

The examples of nonprehensile manipulation given above range from very large ungraspable objects at one end, to many small objects at the other. This work concerns itself with attempting to reliably and quickly orient small objects, although some results may be useful in other domains. We are interested in controlling the shape of constraint surfaces of systems in such a way that constraint and external forces naturally attract the system to the desired state. Palmar manipulation offers a simple domain in which to explore this question.

Systems which perform the type of tasks we will focus on include bowl feeders and Automatic Parts Orienting Systems ([5], [17]), where a large mass of parts in arbitrary orientations are singulated and oriented by their interactions with (in the case of bowl feeders) fences and other obstacles, or (in the case of APOS systems) by an induced vibration and their interaction with pallets of special shapes. Peshkin and others have studied the case of parts along a conveyer belt, with stationary fences along the way to do the orientation ([27], [7]).

For these tasks, we can take advantage of the fact that only the final state needs to be precisely determined. We will show, using an analysis similar to that described by Trinkle, *et.al.* ([32],[33]) that the state space of the manipulator and object can be divided into regions which are equivalent, in the sense that a particular state is easily reachable from one of its equivalent states. Hence, control of the manipulators during the motion need only be precise enough to make the regional transitions correctly, and the object will be propelled along a correct path to the goal.

2 Related Work

One of the earliest examples of robotic nonprehensile manipulation is quasistatic pushing in the presence of fric-

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tion ([22], [8], [19], [26], [7], [20], [3]). This is an example of what could be called *palmar manipulation*, that of one object by one hand. Palmar manipulation will be defined to be the manipulation of an object by its interactions with flat, nearly rigid manipulators of fixed geometry, somewhat analogous to manipulating an object with the palm of a hand, without the fingers. Peshkin [26] works out the minimum distance which a fence (or palm) must push an object so that it will be guaranteed to come to rest aligned with the fence at the end of the motion. Goyal ([15], [16]) shows that the motion of an object on a frictional support surface can be determined if the pressure distribution of the object is known. Peshkin and Sanderson [26] and Lynch [19] analyze this situation when the pressure distribution is not known. Mani and Wilson, Peshkin and Sanderson, and later Brokowski, Peshkin, and Goldberg ([21], [27], [7]) use the mechanics of pushing to design parts orienters or parts filters with a sequence of fences, similar in function to the vibratory bowl feeders and other orienting systems described by Boothroyd, *et. al.* [5].

When pushing a planar lamina, the gravitational and support forces are perpendicular to the plane of interest. Bat juggling ([9]) may also be considered as palmar manipulation of one (or more) objects, where now gravitational forces are present in the plane of interest.

Yun [35] studies two manipulators with open palm end effectors manipulating an object by having one palm push from one end, and another palm push at the other end. The desired effect is to push with both palms hard enough so frictional forces counteract gravitational forces, but not so hard as to damage the object. Coordinated pushing is then used to maneuver the object as desired. This work was extended in [25]. Here, multiple planar palms are used to manipulate large objects in free space. A primary difference between that work and the present effort is the emphasis in the former on force closure. Rolling contacts between the palms and the object are permitted, but the contacts are not allowed to slip or break. Hence, direct control over the object state must be maintained at all times. In this present work, however, the object state is not directly controlled at all times, but merely guided towards the desired goal. This type of manipulation, and indeed much of the work on pushing may be considered "passive" manipulation, in contrast to the more "active" manipulation explored in [35], [25], and in much research on grasping.

Examples of passive manipulation appear in the whole-arm-manipulation techniques of Trinkle, Ram, Farahat and Stiller ([33], [13], [30], [31]). Their analysis and planning use the idea of *contact formations* originally presented by Desai, and incorporated into a planner for dextrous manipulation by Trinkle and Hunter [32]. The work described in the present paper follows a similar method to Trinkle, *et. al.*. However, the paths through configurations space by [33] and [32] are apparently constrained to always correspond to stable grasps, whereas the method described in this paper allows certain unstable motions during the transition from state to state. Abell and Erdmann [1] use a similar method as well to study hand-offs of stably supported objects between sets of frictionless point fingers.

In the area of parts orienting, the work by Brost [8] on the orientation of objects by "squeeze grasps" is analogous to palmar manipulation of one object with two hands, with gravity perpendicular to the plane. He finds sets of actions which reliably orient a part in the presence of uncertainty in the part's location. Goldberg, and later Rao and Goldberg ([14], [28]) present algorithms for determining sequences of squeezes of a parallel jaw gripper which will reliably orient frictionless polygonal and algebraic planar parts from arbitrary and unknown initial orientations, without sensors. Mason and Erdmann [12] use gravity to propel parts onto a flat surface, or into a corner formed by two perpendicular flat surfaces, in such a way that the resulting contact forces reliably orient a part.

In the area of dynamic nonprehensile manipulation, Mason and Lynch [23], [24] are exploring the use of dynamic properties for controlled club throwing. Arai and Khatib [4] studied the exploitation of inertial properties of objects for manipulation of objects in ways other than throwing. They base their approach on work by Arai in using dynamic coupling to control passive joints. Erdmann [10] is also exploring the use of dynamic effects, friction and compliance in nonprehensile palmar manipulation. He studies "slide transfers" of objects from a palm to another planar surface (such as another palm).

3 Frictionless "Cone Manipulation"

For the rest of the paper, we make the following assumptions:

- We will restrict ourselves to planar polygonal objects, although our methods should carry over to any planar object whose convex hull has a finite number of stable resting positions. Cylindrical objects can also be modelled as planar.
- Force balance is achieved by the palms stably supporting the object against a known gravitational force. No other external forces are considered, hence complete force/form closure is not necessary.
- We will assume that the motions of the manipulator are slow compared to gravity, so that the kinetic energy imparted to the object by the motion of the palm is dominated by the object's potential energy.

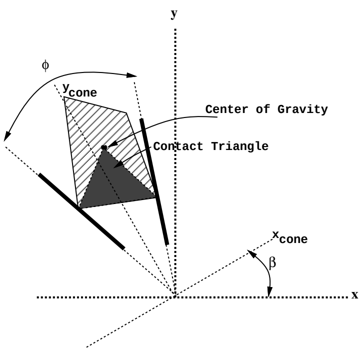

Figure 1: Contact triangles and Cone frames

- We will assume that the contacts between the object and the palms are very low friction (*i.e.*, the contacts are all sliding), so that we may approximate the system with a frictionless analysis.
- We will use the example of a "cone" manipulator: two palms connected at a central hinge. This is a simple example that still captures many of the basic operations of a general two palm system.

To justify the last assumption, we notice that many (though by no means all) configurations of two palms which are capable of passively supporting an object can be modelled by the behavior of the object resting in a cone (Figure 1). This cone is formed by the intersection of the lines along which the palms lie. Set a frame in the cone such that the y axis is the bisector of the cone. Let ϕ be the angle of the cone opening, with the y axis of the cone frame bisecting the opening, and let β be the orientation of the cone frame with respect to the world frame. Then all motions of the cone can be described in terms of the change in ϕ and the change in β.

If an object is in at least one point contact with each of the palms, the resulting motions can be abstracted by the motion of a triangle in a cone, where the "top" vertex of the triangle is the center of gravity (CG) of the object (Figure 1), and the left and right vertices are the points of contact with the left and right palms, respectively. The center of gravity traces out a concave down elliptical curve as the triangle rotates, maintaining two point contact with the cone. If, further, the cone frame is tilted at an angle β counterclockwise with respect to the world frame, then the potential energy of the object is $CG_x \sin\beta + CG_y \cos\beta$. Potential energy minima correspond to stable equilibrium, where the net force on the object is zero, and the object will resist small perturbations of its orientation. Potential energy maxima correspond to unstable equilibrium, where the net force on the object is zero, but the object cannot resist small perturbations of its orientation. There is at most one unstable equilibrium orientation of the triangle balanced in the cone, and one or two stable resting orientations, corresponding to the triangle resting on one side or other of the cone. If we look at all the triangles formed by all possible pairs of vertices of the object, we can generate the potential function curve of the object in a given cone tilted at a given orientation. Figure 3 shows a scaled example of such a curve, for the example object shown in Figure 2, in a cone opened $\pi/3$ radians wide, with $\beta = 0$. Since the local minima of this curve represent the orientations where two triangles are simultaneously in contact with the cone, stable orientations of an object correspond to three point contact with the cone. Let x be the horizontal position of the object CG, and θ by the orientation of the object, both in the world frame. For a fixed value of β, the curve defined by the constraints x and θ imposed by 2 or 3 point contact of the object with the cone forms a "valley" into which the constraint surface gradients point (Figure 3). This valley, in turn, has local minima which attract the system state. Hence, assuming that the palms move slowly in comparison with gravity (so that kinetic energy is low), an object caught in a cone in a particular orientation will settle to a unique resting position determined by the initial position of the object upon contact, and the tilt of the cone with respect to gravity.

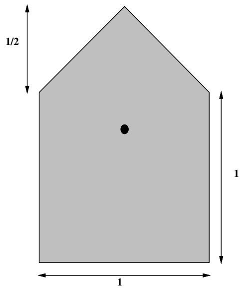

Figure 2: Example Object

Once an object is in a stable state, the cone can be tilted back and forth within a certain range of β while maintaining stability. For those regions of β where stable contact is maintained:

$$\frac{\partial\theta}{\partial\beta} = 1. \tag{1}$$

We now look at what happens as ϕ is varied and β is fixed. If the object is already resting stably in edge contact with one palm, then for some range of ϕ, the cone can be widened or narrowed and the object will stay in stable contact with that palm. For the range of ϕ for which the object maintains stability,

$$\frac{\partial\theta}{\partial\phi} = \pm\frac{1}{2}, \tag{2}$$

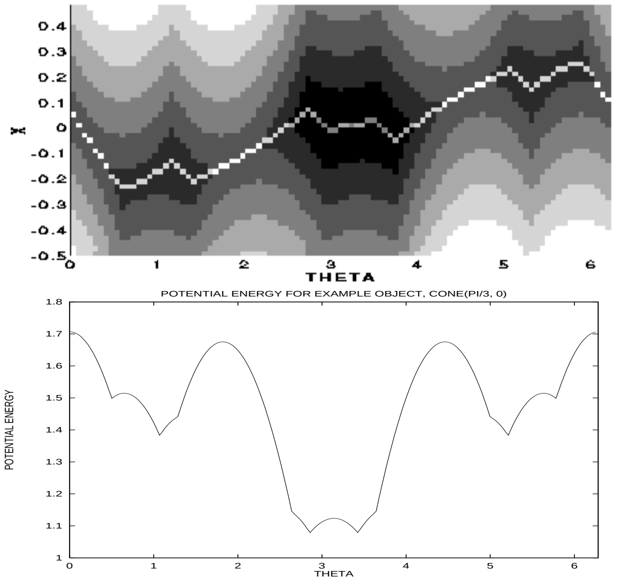

Figure 3: Constraint surface in Configuration Space (θ, x, y), $\phi = \pi/3, \beta = 0$). (darker = lower). Dashed line is 2 point contact "valley", also shown projected into the $(\theta\ ,\ y)$ plane

positive if the object is resting on the left palm, or negative if on the right palm. We will ignore the possibility of jamming, since the contacts are assumed always to slide.

The change in orientation of an object in response to the tilting and squeezing of the cone can be described by the equation

$$\begin{aligned} \partial\theta &= 1 \cdot \partial\beta - \frac{\varepsilon}{2} \cdot \partial\phi \qquad (3) \\ \varepsilon &= -1, \text{ resting on left palm} \\ & \quad\ 1, \text{ resting on right palm} \end{aligned}$$

over the range of ϕ, β for which the object remains stably supported throughout the cone motion.

3.1 Planning in the Frictionless Domain

We would like to use the above observations to plan object reorientations automatically. If we look, for the moment, at the object only after it has made contact with the cone, then the cone/object configuration can be characterized as a point in the space (θ, ϕ, β). We will call all stable resting configurations that correspond to a particular side of the object in edge contact with a particular palm *equivalent configurations*. For example, (see Figure 4), all stable configurations where rectangle side *a* rests on the left palm are equivalent. Referring to Equation (3), we see that a surface of equivalent configurations (the *equivalence region*[1]) in (θ, ϕ, β) space is a planar surface. Figure 5 shows the projection of this surface into the (ϕ, β) plane. We call this projection the *shadow* of the equivalence region. For stable edges, the equivalence regions can be shown to be simply connected [36]: for any two configurations in an equivalence region, there is a stable trajectory from one configuration to another. Hence, if we have the point $(\theta_{des}, \phi_{des}, \beta_{des})$ as our goal state, then an immediate subgoal is to reach the corresponding equivalence region.

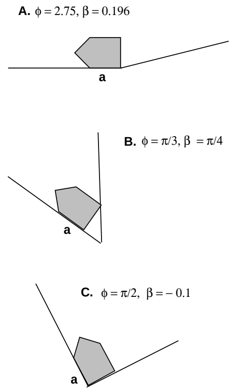

Figure 4: Different configurations which are in the same equivalent region

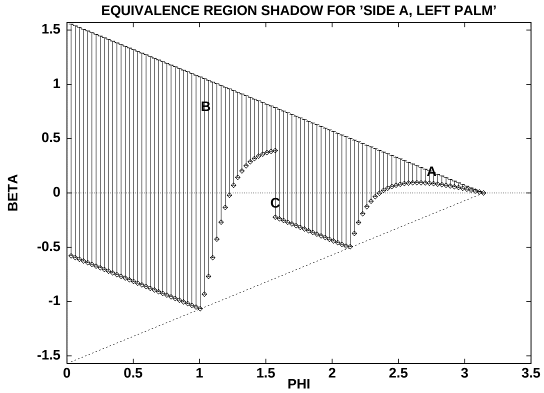

Figure 5: Equivalence region shadow for configurations of previous figure. The labeled points show the locations of the corresponding configurations in the previous figure.

Recall that for the frictionless, low kinetic energy case, every configuration for the object and cone which is in the constraint surface "valley" (except for the unstable equi-

[1] We note here that the interior of what we are calling equivalence regions are supersets of what are called in [29] *passive first order stability cells (passive FS cells)*. Specifically, one of our equivalence regions contains a union of one or more passive FS cells, each one corresponding to a different set of vertex contacts. An equivalence region corresponds to a region in configuration space for which a completely (first order almost everywhere) stable path exists between any two points in the region.

librium orientations) is attracted to a unique stable resting configuration. Then for a fixed cone, every stable object orientation θ_s has a neighborhood of orientations which converge to θ_s. Taking the union of all these neighborhoods in (θ, ϕ, β) space gives the preimage $\mathcal{P}_{\mathcal{S}}$ of an equivalence region, $\mathcal{S}$, defined as the set of all configurations (θ, ϕ, β) which converge to some configuration (θ_s, ϕ, β) such that $(\theta_s, \phi, \beta) \in \mathcal{S}$. In other words, $\mathcal{P}_{\mathcal{S}}$ is a region of state space which is trapped in a potential energy well. The bottom of the energy well is the surface $\mathcal{S}$.

The edges of an equivalence region correspond to the edge of stability of a particular contact class, $\mathcal{S}_0$: there will be some directions of movement of the cone which will take the system state out of $\mathcal{S}_0$, causing the edge contact of interest to be lost. If that particular boundary region of $\mathcal{S}_0$ lies in the preimage of another equivalence class, $\mathcal{S}_1$, then the object will fall into the stable contact corresponding to equivalence region $\mathcal{S}_1$. In other words, an object's orientation can be brought from $\mathcal{S}_0$ to $\mathcal{S}_1$ by bringing the object to the appropriate boundary of $\mathcal{S}_0$ and moving the cone in such a way that the system state moves out of $\mathcal{S}_0$, and into the preimage of $\mathcal{S}_1$. This transition is reliable even though the manipulator does not maintain stable support of the object during the transition from $\mathcal{S}_0$ to $\mathcal{S}_1$, as long as the object's kinetic energy is low compared to its depth in the potential energy well.

In order to determine which orientations of a particular part can be brought to which other orientations:

1. First, determine all the equivalence regions, (two for every flat face of the convex hull of the object) and their preimages.

2. Determine the boundary of each equivalence region, and divide each boundary into segments, according to which new equivalence region that segment can transit to, if any, and what motion of the cone must be used to achieve this transition.

3. Construct the graph $\mathcal{G}$ whose nodes are the equivalence regions, with arcs denoting which equivalence regions transit into another. Each arc is labeled with the appropriate set of cone configurations, and the direction in cone configuration space in which the cone must be moved. Figure 6 shows $\mathcal{G}$ for our example object. The arcs in $\mathcal{G}$ were determined by using only pure tilts (changing β while holding ϕ fixed) at the equivalence region boundaries to transit from region to region. Different contact formations in each equivalence region are shown with an arc into the contact formation of another region which it transits to.

The graph $\mathcal{G}$ in Figure 6 was generated assuming that all configurations (ϕ, β) are achievable. In reality, not all cone tilts and orientations will be achievable, because the finite length of the palms will prevent the object from being held in some cone configurations, and because of other physical limitations of the device. As shown in Section 4, this can cause certain arcs of $\mathcal{G}$ to be completely eliminated.

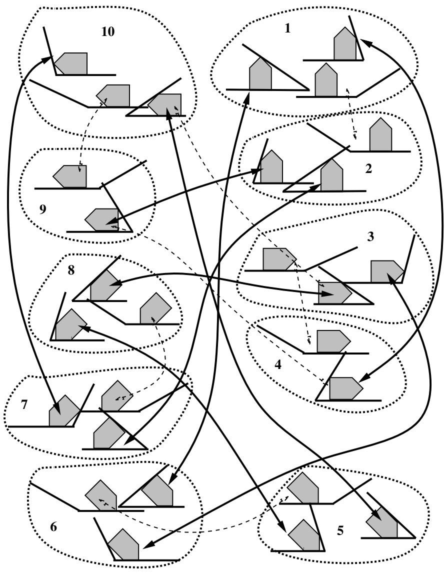

Figure 6: Transition graph for example object. Transition motions are made by pure tilts in the appropriate direction.

The planning problem has now been segmented into two parts. Given the initial and desired final configurations of the system, the high level problem is how to get from the initial to the final equivalence region. This can be determined by straightforward graph search on $\mathcal{G}$. If a path through the graph exists, the reorientation is in principle possible, and the path determines a series of sets of equivalence regions which the system trajectory must go through.

Once it has been established that a high level path exists, the lower level trajectory planning problem for each equivalence region (node) is to determine the trajectory which the cone must follow to reorient the part. The motions to transit from one equivalence region to another are given by the arcs of $\mathcal{G}$. To determine trajectories through equivalent regions, we can take advantage of the fact that equivalence regions are piecewise straight-line connected, as is shown in [36]. Figure 7 shows an example reorientation for our example object.

4 Experimental Results

The preceding algorithm was implemented in C on a Dec-station 5000/20. For the example object, the transition graph $\mathcal{G}$ (Figure 6) can be generated in about one minute. Once $\mathcal{G}$ is generated, reorientation plans can be found in one or two seconds. Plans were generated to bring the

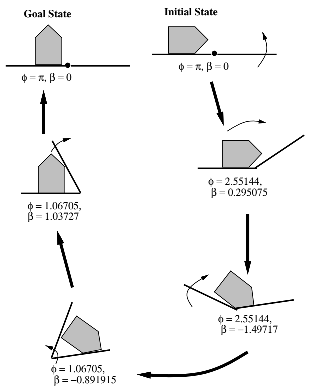

Figure 7: Example reorientation: from $\theta = -\pi/2$ on left palm to $\theta = 0$ on left palm

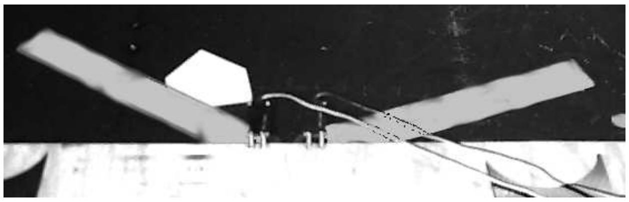

Figure 8: Plastic cone manipulator used to test plans

object from the initial stable orientation on a flat palm, to a goal stable orientation on the goal palm, much as in Figure 7. The plans were tested both in simulation and on a plastic cone manipulator (Figure 8), mounted on a tilted air table to reduce support friction. Note that the transition graph in Figure 6 was generated assuming that all feasible cone configurations are achievable. To generate plans which could be executed by the manipulator, motions corresponding to the object sliding from one palm to the other over the central hinge point, and configurations where $\phi < 0.5$ were disallowed, and $\mathcal{G}$ regenerated. This resulted in certain arcs being completely eliminated from the graph, and they are shown as dashed arrows in Figure 6.

For the example object, the plans simulated tended to either be "robust" to friction as high as about $\mu = 0.25$, or be extremely sensitive to the frictionless approximation, failing for friction higher than $\mu = 0.02$. Simulation of the plan shown in Figure 7 showed that a static coefficient of friction $\mu \leq 0.25$ would permit enough of the contacts to slide for the predictions from the frictionless approximation to be valid, and for the plan to succeed. In the experiments conducted on the air table, the static coefficient of friction was approximately 0.19, low enough for the "robust" plans to succeed. To evaluate the reliability of the example plan, we ran 50 trials, starting the object in its initial orientation, $\phi = -\pi/2$, at different arbitrary locations on the left palm. The varying initial configuration of the part led to variation of the part's trajectory through its configuration space, as expected. Nonetheless, of the 50 attempts, the manipulator failed to correctly reorient the object only 4 times. Despite the variation in the object's trajectory, the object always stayed trapped in the correct region of the state space and hence would be propelled along to the correct final orientation. Each of the failures seemed to be due to a single rough spot on the right palm, which caused a contact to roll rather than slide.

5 Conclusion

We have presented a preliminary analysis of nonprehensile manipulation by two low friction palms, and developed a planning method for part reorientation with our model. Our method finds feasible paths through the space of equivalent state configurations of the object in the palms, without requiring that the palms maintain stable support of the object over the entire path. We have implemented our planner, and checked the plans both in simulation and physically. Early results are encouraging. Future work will focus on relaxing the requirement that all contacts slide, in order to extend the model to systems with higher coefficients of contact friction. Also of interest is finding plans for orienting objects reliably to known goal states from uncertain initial states, as in the grasping plans of [14], [28].

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