Angular Momentum Versus Mass, Re-Examined

A plot of angular momentum vs. mass for a wide variety of astronomical objects and systems is well fit by $J \propto M^b$ and $b = 1.9$. Some authors have seen in this the working of a new physical principle that might be used to help understand the origin and evolution of structure in the universe. It is concluded here that the relationship can be fully understood using Newtonian mechanics and other familiar physics. A random sampling of terrestrial objects yields an equally tight (and equally comprehensible) relation with $b = 1.5$. It seems unlikely that further consideration of plots of J vs. M can yield new clues to the formation of galaxies, stars, or anything else.

When you don't understand a phenomenon very well, the first thing to do is to plot it on log–log paper.[1] And the phenomena astronomers understand least well today are surely formation processes—for galaxies, globular clusters, stars, planets, comets, and all the rest. Brosche[2] seems to have been the first to suspect that some insight into origins might come from examining angular momenta for a wide variety of objects as a function of their masses. He plotted log J vs. log M for 12 spiral galaxies, nearby double stars, and six planet–satellite systems, with estimates for clusters of stars and galaxies.

The surprising result was that a single straight line, with slope two ($J \propto M^2$) fit all the points and limits, with deviations very small compared to the enormous range in both variables covered by the plot. He proposed no particular explanation, but suggested that one would be needed and that it would be likely to be interesting.

Many later authors have confirmed Brosche's result,

$$\log J = \log J_0 + b \log M; \; b = 1.5\text{–}2.0, \tag{1}$$

Comments Astrophys.
1984, Vol. 10, No. 4, pp. 127–135
0146–2970/84/1004–0127/$20.00/0

TABLE I
Angular momentum vs. mass for astronomical objects and systems

Object	log M	log J	log J_{max}/constraint	log J_{min}/constraint	Source of data
Comet	15.0	20.2	21.5 rotational instability	18.5 spin-up by gas ejection	Ref. 21
Asteroid	21.2	29.9	31.3 rotational instability	26.0 phase-lock to orbit period	Refs. 21, 4
Moon	25.9	35.1	38.8 rotational instability	35.1 phase-lock	Ref. 4
Earth	27.8	40.8	42.2 rotational instability	38.1 phase-lock	Ref. 22
Earth–Moon	27.8	41.5	42.3 tidal disruption by sun	40.7 Roche limit	Refs. 22, 4
Jupiter	30.3	45.4	46.0 rotational instability	41.3 phase-lock	Ref. 4
Sun	33.3	47.3–49.0	50.0 rotational instability	47.0 wind turn-off	Refs. 23, 4
Solar system	33.3	50.3	52.5 escape of planets	49.3 Roche limit	Ref. 4

Binaries (Ecl.)	34.0	52.7	55.7 disruption by other stars	51.8 contact	Refs. 18, 19
Binaries (Spect.)	34.0	53.0	55.7 disruption by other stars	51.8 contact	Ref. 19
Globular cluster (M3)	39.1	62.9	63.5 rotational instability	62.2 phase-lock	Ref. 24–27
Globular cluster (ω Cen)	39.7	63.9	64.8 rotational instability	62.8 phase-lock	Refs. 24–27
d Sph orbiting *M*31	41.8	68.8	69.7 unbinding	68.6 phase-lock	Refs. 3, 4
Milky Way (disc)	44.3	73.8	74.0 rotational instability	71.0 phase-lock	Ref. 3
Giant elliptical	45.3	< 72.5 – 73.4	74.9 rotational instability	— none for isolated object?	Ref. 28
Local group	45.7	76.0	76.6 unbinding	73.6 contact	Refs. 3, 4
Supercluster	48.3	78.6	80.4 break-up	77.3 $\sqrt{N}/N$ excess of orbits	Refs. 3, 4
Observable universe	55.5	91.1	95.0 inhibition of galaxy formation	— none?	Ref. 8

and it can be recovered by anyone with a copy of Allen's *Astrophysical Quantities*[3] or a good elementary astronomy text[4] and a sharp pencil. My version is represented by the first three columns of Table I and the dots in Figure 1. A formal least-squares fit (giving equal weight to comet, asteroid, earth, moon, Jupiter, solar system, binaries, average globular cluster, dwarf spheroidal galaxy, Milky Way, giant elliptical galaxy, local group, and supercluster) yields $b = 1.91$, $\log J_0 = -11.65$. The form of the result is quite insensitive to whether or not the sun has a rapidly rotating core, whether you count the earth in isolation or the earth–moon system, whether the outer Milky Way has a flat or Keplerian rotation curve, and so forth.

What does it all mean? Muradian,[5–7] especially, has drawn sweeping conclusions. He uses the form of J vs. M as support for a model of universal evolution (originally proposed by V.A. Ambartsumyan) in which existing structures have developed from the expansion and fragmentation of super-dense, neutron-rich matter. Sisteró[8] draws similar conclusions. And Sternglass's[9] relativistic electron-pair model of matter, while very different in conception, also implies a relation like Eq. (1) and is thus, in some sense, supported by it. These authors also tie some of the Dirac "large numbers"[10–12] to the J–M relation. One result is a prediction of the angular momentum of the entire observable universe. It is quite close to the value implied by rotation measures of extragalactic radio sources.[8,13,14] The required rotation period is near 10^{13} yr and just misses violation of observed upper limits to the anisotropy of the microwave background radiation.[15,16]

In contrast to these very broad conclusions, Carrasco *et al.*[17] looked at angular momentum vs. mass for spiral galaxies and decided that one could, in this way, rediscover Kepler's third law by a very difficult method. They found that no additional physics was necessary to explain the relation, given the observed range of galaxy sizes.

I first met the problem through binary stars. About 1000 eclipsing systems are well fit by $J \propto M^{1.8}$.[18] But this is apparently not profound. It is exactly what you expect for contact systems, obeying Kepler's laws, and constrained to have the stars separated by the sum of their radii, given the known mass-radius relation for main sequence stars.[19] Most spectroscopic binaries, in contrast, are not contact systems, and 978 of them are best fit by $J \propto M^{5/3}$—just what you expect for a population that has a limited range of periods and obeys Kepler's law.[19]

To summarize thus far, there seems to be considerably less than

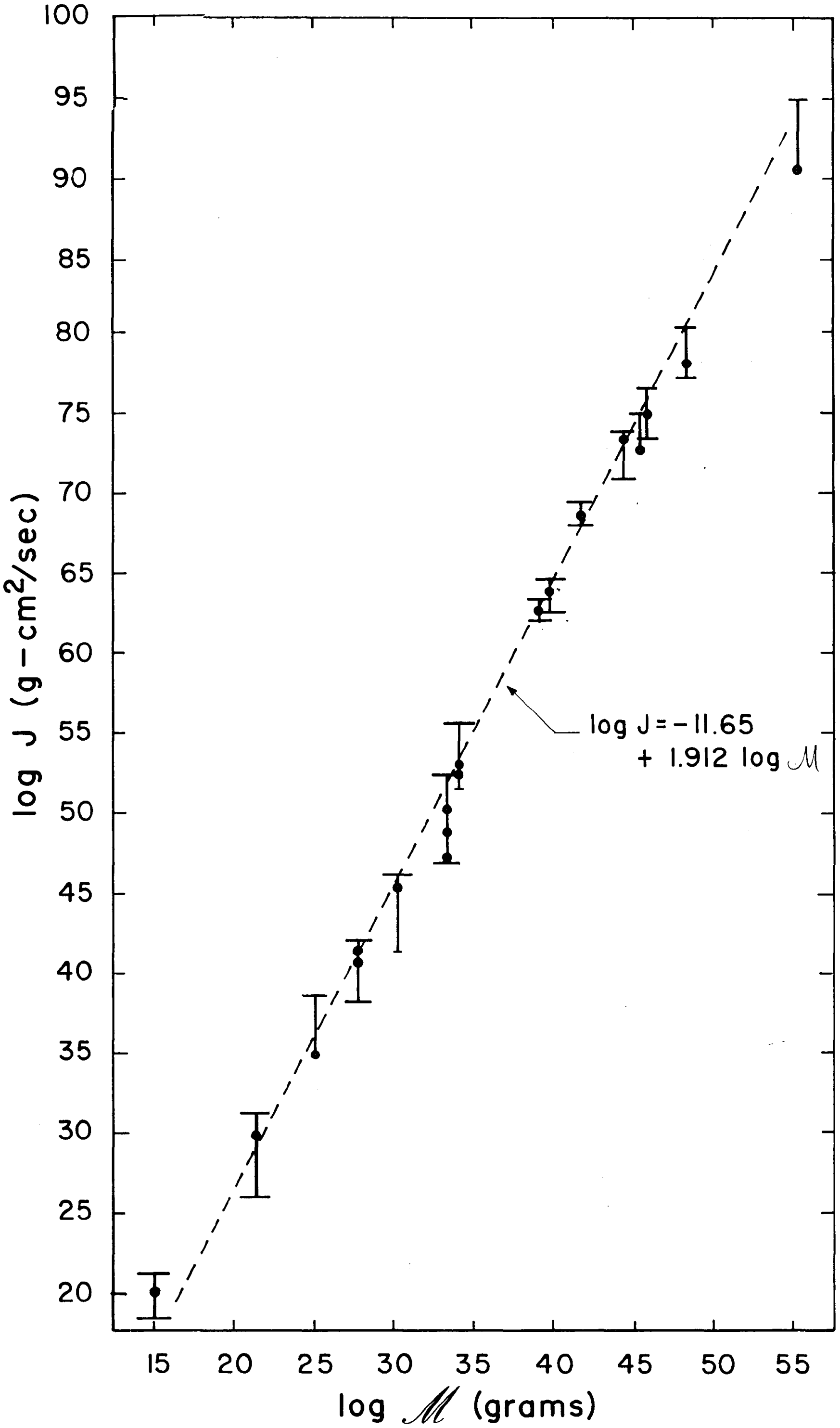

FIGURE 1 Angular momentum vs. mass for the astronomical objects and systems listed in Table I. Dots are actual values of J, error bars the full range of values permitted to the object. The dashed line is a formal, least-squares fit to the actual values, but any line drawn through the permitted values will be rather similar. Thus J vs. M does not seem to be an independent datum about astronomical objects.

meets the eye to the form of Eq. (1) for spiral galaxies and binary stars. The next natural question is then whether other portions of the universal *J–M* curve might not have equally prosaic explanations.

Let's figure out for each of the objects or classes of objects in Table I what are the physically possible maximum and minimum angular momenta. Columns 5 and 6 give these numbers with two-word descriptions of the physics that sets the limit. The last column contains the sources of the data used.

Most of the physical processes are obvious, and nearly all are dominated by gravitational interactions—tidal disruption or locking of rotational to orbital period when things get too close to each other; unbinding of the system by interactions with other objects when they get too far apart; dynamical instabilities when things try to rotate too fast; and so forth. Among the less obvious ones, comets and asteroids are spun up by asymmetric gas ejection and collisions respectively. Solar type stars begin life as relatively rapid rotators and are slowed down by magnetized winds that, apparently, turn off when the rotation becomes too slow to sustain a significant dynamo field. We suspect, from the weakness of the solar wind and from rotational velocities of other stars as a function of mass and age, that our sun is rather close to this turn-off. J_{min} for a rich cluster of galaxies or supercluster is set by a $N^{1/2}/N$ excess of orbits going in one direction, within a velocity distribution just anisotropic enough to reproduce the observed, flattened shape. It goes up if some of the flattening of such structures is rotationally induced. I can think of no reason why a completely isolated elliptical galaxy should have to rotate at all; but it is not clear that there are any truly isolated galaxies.

For the universe as a whole, J_{max} is such that the resulting shears would be likely to inhibit galaxy formation.[15] A faster-rotating universe is physically possible, but this journal would not be published in it. I can think of no reason why J_{min} should not be zero. The real value will be exceedingly small in a universe that has passed through an inflationary phase.[20] The nonrandom distribution of radio source rotation measures would have then to be attributed to improper subtraction of the contribution from our own interstellar medium. The location of the pole of the anisotropy, near the direction of the maximum of the integrated line-of-sight electron density, supports this alternative interpretation. The "universe" point should then be removed from both figure and table.

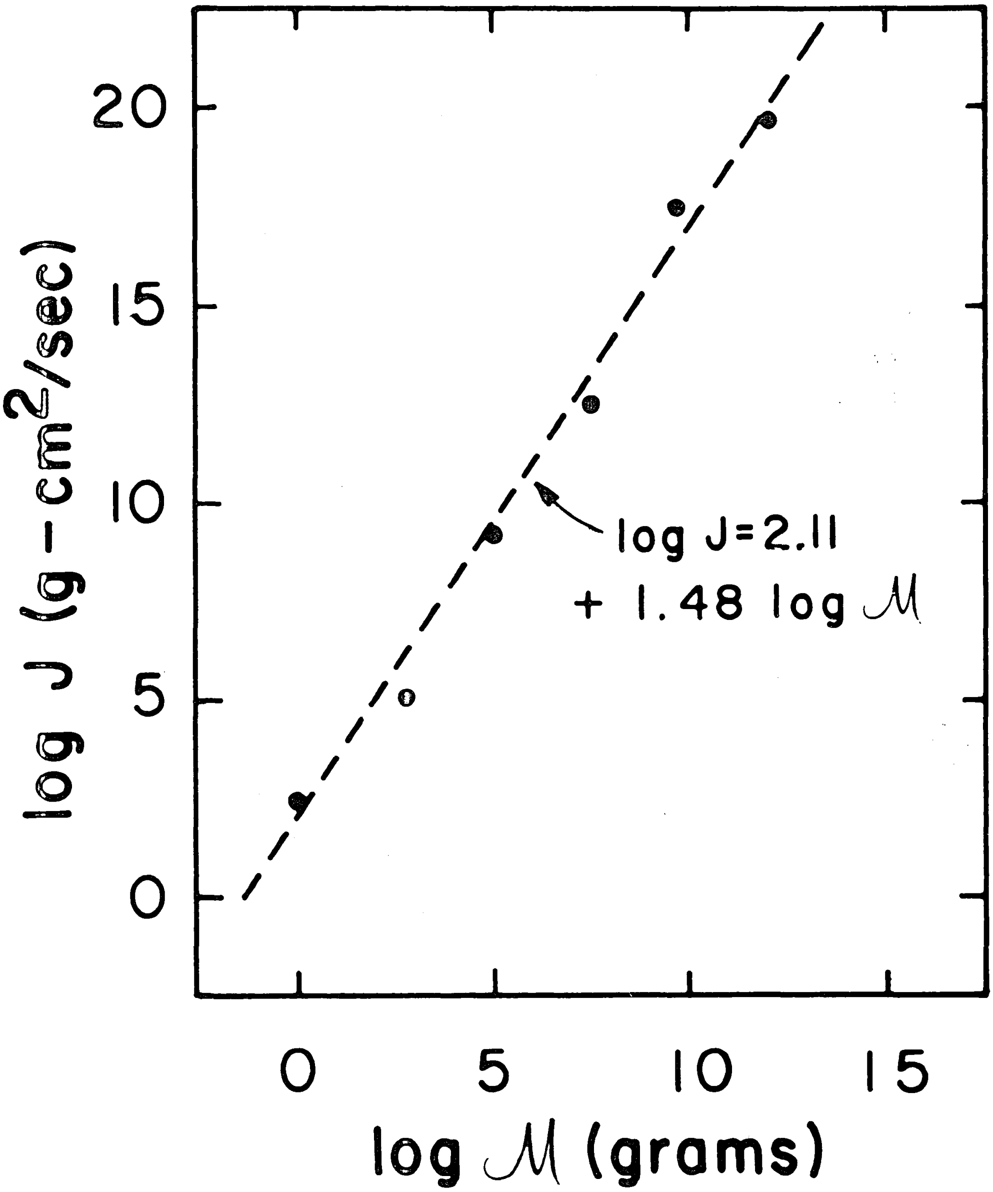

FIGURE 2 Angular momentum vs. mass for common terrestrial objects. Dots are actual values of J and the dashed line a least-squares fit to them. Most have close to the maximum angular momentum permitted before fracture or slippage would occur, and the relation seen, therefore, depends only upon the strengths of solid body and frictional forces.

Look again at Figure 1. The things resembling errors bars are, in fact, the full range of possible values of J for each object, as determined from these assorted dynamical considerations. Any straight line you care to draw through the permissible ranges will clearly have very nearly the same slope and zero point as the best fit to the actual values.

The implication is that J vs. M for astronomical objects is largely a result of their structure being dominated by gravitation and the actual values of their masses and densities. Some of these numbers, like the masses and sizes of stars; can be calculated and others, like galaxy masses, plausibly estimated[12] from the balance between gravity and pressure and other well-known kinds of physics. If the form of Eq. (1) can be understood using only familiar principles, it cannot properly be used as evidence for anything revolutionary, or even as an independent datum to be matched by models for star formation, galaxy formation, etc.

By way of analogy, Figure 2 presents angular momentum vs. mass for six common classes of terrestrial objects, like Ferris wheels, footballs in play, ships coming about, etc. They were chosen to have values of log M spaced at intervals near 2.5, from a fly at $M = 1$ g to a newly calved iceberg at 10^{12} g. A least squares fit yields $b = 1.48$, log $J_0 = 2.11$, with rather less scatter than for the astronomical objects. It is left as an exercise for the reader to persuade himself that such a relationship is just what we would expect for objects whose densities, strengths, and frictional forces are all dominated by electromagnetic forces (rather than gravitational ones, as for the astronomical objects), when they have J's fairly close to the maxima at which slippage or fracture would occur. J_{min} is zero for these objects simply because we can imagine exerting physical or moral forces that will transfer their angular momenta to the earth without perceptably spinning it up.

This terrestrial analogy is meant as a warning against (a) uncritical use of log–log paper and (b) attributing exotic causes to things explicable by well-known pieces of physics.

Acknowledgments

I am indebted to L.S. Trimble, Joe Weber, and an anonymous UC Irvine chemistry student for sharing with me their knowledge of Ferris wheels, aircraft carriers, and footballs, respectively.

VIRGINIA TRIMBLE
Department of Physics,
University of California,
Irvine California 92717
and Astronomy Program,
University of Maryland,
College Park, Maryland 20742

References

1. P. A. G. Scheuer, Discussion on early history of radio astronomy in Cambridge (1968).
2. P. Brosche, Z. Astrophys Ap. **57**, 143 (1963).
3. C. W. Allen, *Astrophysical Quantities* (London, Athlone Press, 1973).
4. G. O. Abell, *Exploration of the Universe,* 4th ed. (Saunders, Philadelphia, 1982).
5. R. M. Muradian, Astrofizs. **11**, 237 (1975).
6. R. M. Muradian, Astrofizica **13**, 63 (p. 32 in translation) (1977).
7. R. M. Muradian, Astrophys. Sp. Sci. **69**, 339 (1980).
8. R. F. Sisteró, Astrophys. Lett. **23**, 235 (1983).
9. E. J. Sternglass, Bull. Amer. Astron. Soc. **15**, 937 (1983).
10. P. A. M. Dirac, Nature **139**, 323 (1937).
11. P. A. M. Dirac, Proc. Roy. Soc. **165A**, 199 (1938).
12. B. J. Carr and M. J. Rees, Nature **278**, 605 (1979).
13. E. J. Sternglass, Talk at 163rd meeting of AAS (1984).
14. P. Birch, Nature **298**, 451 (1982).
15. C. B. Collins and S. W. Hawking MNRAS **162**, 307 (1973).
16. G. F. Smoot, Phys. Scripta **21**, 619 (1980).
17. L. Carrasco, M. Roth and A. Serrano Astron. Astrophys. **106**, 89 (1982).
18. R. F. Sisteró and S. Marton, Astrophys. Sp. Sci. **94**, 165 (1983).
19. V. Trimble, Astrophys. Sp. Sci. (in press).
20. A. Guth, Proceedings of 1982 Texas Symposium, Ann. NY Acad. Sci. (in press).
21. F. Whipple, in IAU Colloq. **61**, *Comets,* edited by C. L. Wilkening (University of Arizona Press, 1982), p. 227.
22. R. C. Weast and M. J. Astle, *CRC Handbook of Chemistry and Physics* (CRC Press, Boca Raton, 1982), p. F-154–56.
23. M. Schwarzschild, *Structure and Evolution of the Stars* (Princeton University Press, Princeton, 1958), Table 28.3.
24. J. E. Gunn, in *Globular Clusters,* edited by D. Hanes and B. Madore (Cambridge University Press, Cambridge, 1980).
25. K. C. Freeman and P. Seitzer, unpublished work cited by Refs. 26 and 27 (1980, 1983).
26. S. van den Bergh, Publ. Astron. Soc. Pac. **95**, 839 (1983).
27. K. C. Freeman and J. Norris, Ann. Rev. Astron. Astrophys. **19**, 319 (1981).
28. G. Illingworth, in *Structure and Evolution of Normal Galaxies,* edited M. Fall and D. Lynden-Bell (Cambridge University Press, London, 1981), p. 7.